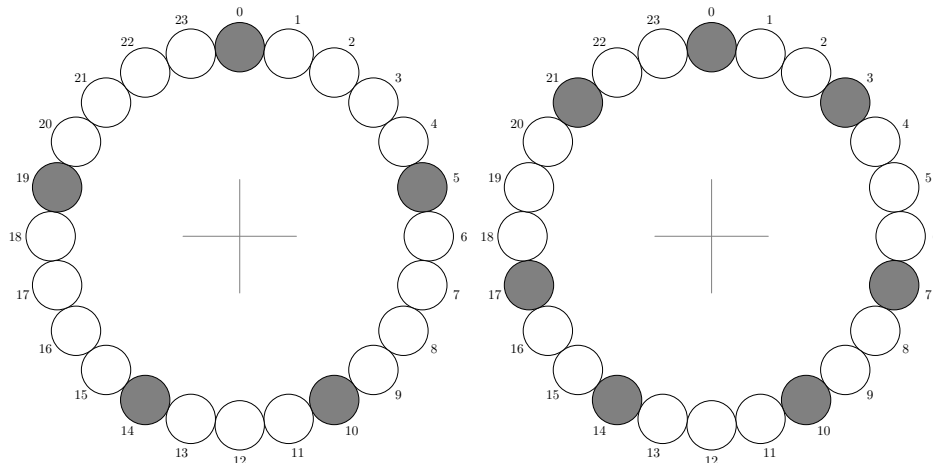


On Badly Balanced Centrifuge Configurations

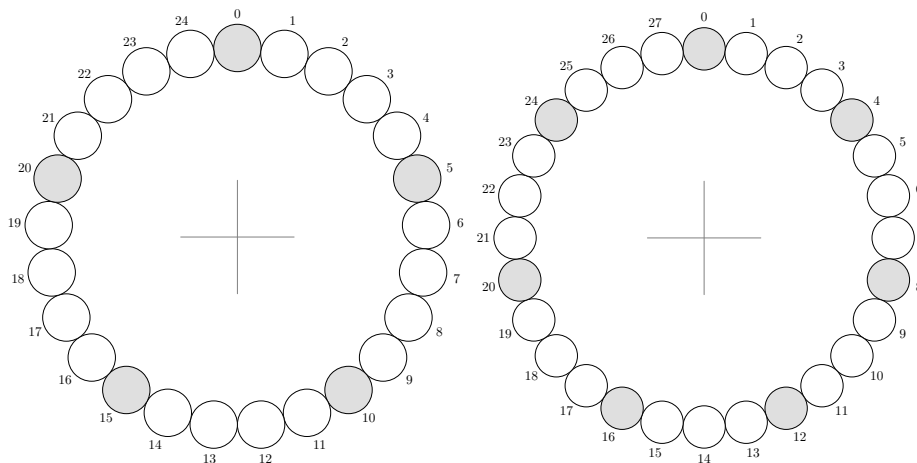
Rob Hubbard

A high-speed centrifuge can be so sensitive to balance that you should weigh the samples to ensure that they are equal (within tolerance), and load them symmetrically, or in symmetrical combination.

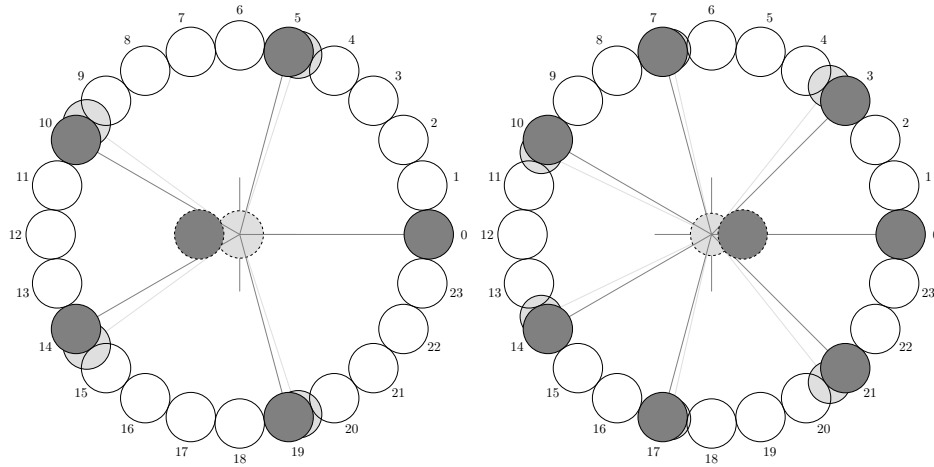
In a certain manufacturer's instructions for their 24-position centrifuge, the following configurations were suggested for 5 and for 7 samples. These seemed dangerously wrong to me.



These, respectively, look superficially similar to a notional 25-position balanced configuration of 5 samples, and a 28-position balanced configuration of 7 samples:



Overlaying the configurations, presenting them in a positive orientation for the complex plane, and showing the effective centres of gravity, we see that the manufacturer-suggested configurations are badly far from being balanced.



A calculation for how far off the manufacturer's configuration of 5 is:

$$\begin{aligned}\zeta_{24} &= \text{cis}\left(\frac{\tau}{24}\right) \\ &= \frac{(\sqrt{3}+1)\sqrt{2}}{4} + i \frac{(\sqrt{3}-1)\sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}s_5 &= \zeta_{24}^0 + \zeta_{24}^5 + \zeta_{24}^{10} + \zeta_{24}^{14} + \zeta_{24}^{19} \\ &= \frac{2 + \sqrt{6} - 2\sqrt{3} - \sqrt{2}}{2} \\ &\approx -(0.2144127^+) \\ |s_5| &\approx 0.2144127^+\end{aligned}$$

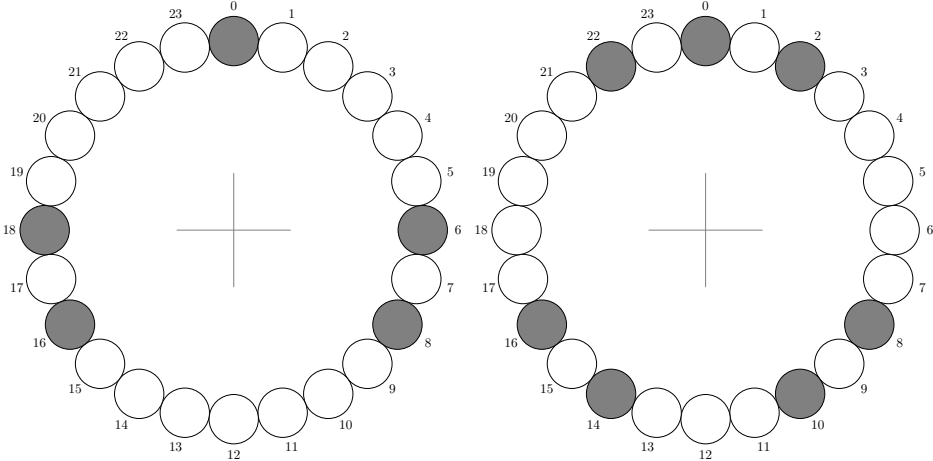
We see that it's over 21.4% of a sample away from being balanced.

A calculation for how far off the manufacturer's configuration of 7 is:

$$\begin{aligned}s_7 &= \zeta_{24}^0 + \zeta_{24}^3 + \zeta_{24}^7 + \zeta_{24}^{10} + \zeta_{24}^{14} + \zeta_{24}^{17} + \zeta_{24}^{21} \\ &= \frac{2 - \sqrt{6} - 2\sqrt{3} + 3\sqrt{2}}{2} \\ &\approx 0.1645247^- \\ |s_7| &\approx 0.1645247^-\end{aligned}$$

We see that it's not quite so bad as the 5-configuration, but it's still over 16.4% of a sample away from being balanced.

Examples of correct, balanced configurations are:



Here, there are groups of two samples balanced in opposite positions, and groups of three samples balanced in a equilateral triangle.